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ANALOGUE MODELING OF THE DYNAMICS OF A MAGNETIC DIPOLE IN AN INHOMOGENEOUS RESONANT MAGNETIC FIELD

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At present, much attention is paid to the problems of the motion of spin particles in electromagnetic fields [1, 2]. The difficulty of solving this class of problems is due to the nonlinearity of the differential equations of motion in partial derivatives [3].

In this paper, we consider the simplest classical model of a spin particle - a magnetic arrow on a rod - and analyze its motion in an inhomogeneous magnetic field. Modeling was carried out on the analog-digital computing complex "Rusalka" [4].

We place a dipole - an arrow with a magnetic moment μ - in an inhomogeneous constant $\mathbf{H} \simeq [0, 0, H(x)]$ and high-frequency magnetic field $\mathbf{H}_1 \simeq [H_1 \cos \omega t, 0, 0]$ transverse pumping.

Let us write expressions for kinetic, potential energies and dissipative functions in the forme

$$T = \frac{I}{2}\dot{\theta}^2 + \frac{m}{2}\dot{x}^2, \quad (1)$$

$$U = -(\mu \mathbf{H}_\Sigma) = -\mu H \cos\theta - \mu H_1 \sin\theta \cos\omega t, \quad (2)$$

$$F = \frac{a_1}{2}\dot{\theta}^2 + \frac{a_2}{2}\dot{x}^2. \quad (3)$$

Types of designation

$$x_1 = \theta, \quad x_2 = \frac{x}{|x_{\max}|}, \quad \beta_1 = \frac{a_1}{2I}, \quad \beta_2 = \frac{a_2}{2m}, \quad \omega_L^2 = \frac{\mu H_0}{m x_{\max}^2}, \quad \omega_s^2 = \frac{\mu H_0}{I},$$
$$\omega_b^2 = \frac{\mu H_1}{I}, \quad H = H_0 h(x_2) = H_0 (1 - k x_2^n) \quad (4)$$

and using the Lagrangian formalism, we obtain the equations of motion

$$\ddot{x}_1 + 2\beta_1 \dot{x}_1 + \omega_1^2 x_1 = \omega_3^2 \cos\omega t,$$
$$\ddot{x}_2 + 2\beta_2 \dot{x}_2 + \omega_2^2 x_2 = 0, \quad (5)$$

where the frequencies $\omega_{1,2,3}(x_1, x_2)$

$$\omega_1^2 = \omega_s^2 h(x_2) \frac{\sin x_1}{x_1},$$

$$\omega_2^2 = -\omega_L^2 \frac{1}{x_2} \frac{dh(x_2)}{dx_2} \cos x_1,$$

$$\omega_3^2 = \omega_b^2 \cos x_1. \quad (6)$$

Let us investigate the possibility of the existence of stable states of motion near the points $(0, 0)$, $(\pm\pi/2, x_{20})$.

Using Sylvester's criterion [5] for the case $\beta_1 = \beta_2 = \omega_b = 0$, we can obtain the stability conditions for stationary states

$$U'_{x_1} = U'_{x_2} = 0,$$
$$U''_{x_1 x_1} > 0, \quad U''_{x_2 x_2} > 0,$$
$$U''_{x_1 x_1} U''_{x_2 x_2} - (U''_{x_1 x_2})^2 > 0. \quad (7)$$

The point (0, 0) satisfies these conditions for the case k> 0, H_0> 0, *n = 2m*, and *U* has a minimum (-μH_0), and at the point (± π/ 2; 0) there is a saddle U_{x1x2}.

Let us show that near the points (± π/2; x_{20}), stable states of motion in the dynamic regime can arise. We use the asymptotic expansion of the functions $\omega_{1,2,3}(x_1; x_2)$ (6) at the points (± π/2; x_{20}) and, leaving only linear terms in (5), we obtain

$$\ddot{\chi}_1 + \omega_{S0}^2\chi_2 + \omega_0^2 = 0, \quad \ddot{\chi}_2 + \omega_{L0}^2\chi_1 = 0, \tag{8}$$

where $\chi_1 = x_1 - \pi/2,\ \chi_2 = x_2 - x_{20},$ and frequencies

$$\omega_0^2 = \omega_S^2(1 - kx_{\max}^n), \quad \omega_{L0}^2 = -\omega_L^2 nkx_{\max}^{n-1}, \quad \omega_{S0}^2 = -\omega_S^2 nkx_{\max}^{n-1}. \tag{9}$$

Then one of the particular solutions for $\omega^2 = \omega_{L0}^2 = \omega_{S0}^2$

$$\chi_1 = \chi_0 \cos\omega t, \quad \chi_2 = \chi_0 \cos\omega t + \chi_{20}, \tag{10}$$

Where $\chi_{20} = -\omega_0^2/\omega_{S0}^2$. Solution (10) is of fundamental importance for problems of particle motion in fields with $c\ n < -2$ of the dipole type. At angles $\theta \simeq \pi/2$, the nature of the interaction changes: attraction $\theta < \pi/2$ and repulsion $\theta > \pi/2$. Pumping occurs energy of translational motion into rotational, and vice versa. The result is the possibility of occurrence of: stable states of motion: and the absence of collapse in the dipole case.

The system of equations (5) was simulated on the ACVK "Rusalka". Taking a constant inhomogeneous field to vary according to the quadratic law

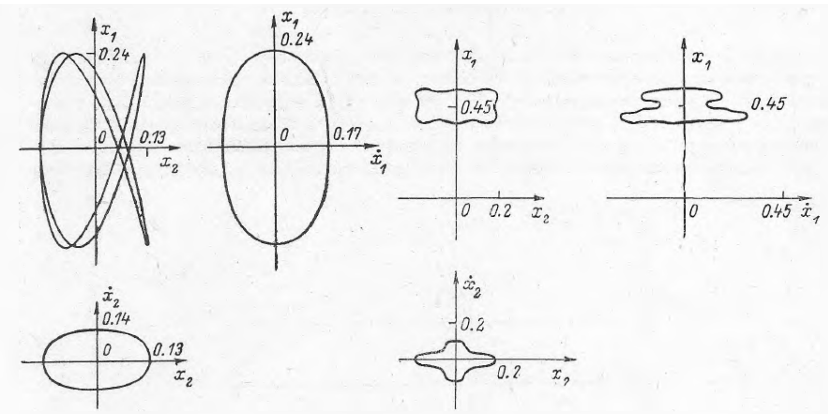
$$h(x_2) = 1 - x_2^2/2, \tag{11}$$

We rewrite system (5) as

$$\begin{aligned} \ddot{x}_1 + 2\beta_1\dot{x}_1 + \omega_S^2(1 - x_2^2/2)\sin x_1 &= \omega_b^2 \cos x_1 \cos\omega t, \\ \ddot{x}_2 + 2\beta_2\dot{x}_2 + \omega_L^2 x_2 \cos x_1 &= 0. \end{aligned} \tag{12}$$

At the first stage of modeling, the dynamics of the dipole was investigated without dissipation and external disturbance, i.e., at $\beta_1 = \beta_2 = \omega_b = 0$. As a result of the simulation, it was possible to determine the numerical values of the frequencies ω_S^2 и ω_L^2, at which stable stationary states of motion are formed. So, for example, when $\omega_S^2 = 0.3$ and $\omega_L^2 = 0.5$ x_1 and x_2 perform harmonic oscillatory motion around the center (0, 0) with a frequency ratio of 1:1, 2:3, 4:7, etc., depending on the initial conditions (Pic. 1).

At the second stage, the dynamics of the dipole was simulated for the case of a perturbed state, i.e., for $\beta_1 \simeq \beta_2 \simeq 0$ and $\omega_b^2 \neq 0$. In the course of modeling, it was found that at small amplitudes of the disturbance, the system forms stable motions around the center (0, 0) with proper frequencies not equal to the frequency of the disturbing force. At large amplitudes of the disturbing



Picture: 1. Stable trajectory, phase portraits with limit cycles for a stationary state.
Frequency ratio 2:3. Initial conditions: $\dot{x}_{10} = -0.11,\ x_{10} = 0.2,\ \dot{x}_{20} = -0.07,\ x_{20} = 0.11.$

Picture: 2. Stable trajectory, phase portraits with limit cycles for a perturbed state.
Initial conditions: $\omega \simeq 0.1\omega_{x_1},\ \dot{x}_{10} = -0.07,\ x_{10} = 0.2,\ \dot{x}_{20} = -0.07,\ x_{20} = 0.11.$

force, the system forms stable trajectories of motion around the points (± π/2; 0) (Pic. 2). When the frequency of the disturbing force is close to the natural frequencies of the system, the value of x_1 increases without limitation, i.e., the arrow twists.

Taking into account the translational and rotational degrees of freedom in a system of spin particles together leads to the equivalent problem of coupled resonators [6]. Accordingly, resonant stable solutions appear [6], which was confirmed in this work.

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